Combinatorics

Mid-Term Examination

Instructions: All questions carry equal marks.

1. Prove that a $k \times n$ Latin rectangle with $k < n$ can be extended to a Latin square of order n.

2. Define a linear space. Prove that the number of blocks in a non-trivial linear space is at least the number of points.

3. If N is the incidence matrix of a symmetric $2 - (v, k, \lambda)$ design, then prove that N^t is also the incidence matrix of a $2 - (v, k, \lambda)$ design (the dual design)

4. Define a strongly regular graph $SRG(v, k, \lambda, \mu)$. If A is the incidence matrix of such a strongly regular graph, then prove that

$$A^2 + (\mu - \lambda)A + (\mu - k)I = \mu J$$

where, I and J denote the identity and the all one matrix respectively.

5. Define a *circulation* on a diagraph D. Prove that if f is an integral circulation on D and if $d \in \mathbb{N}$, then f can be written as the sum of d integral circulations g_i for $1 \leq i \leq d$ such that for every i, we have

$$[\frac{f(e)}{d}] \leq g_i(e) \leq [\frac{f(e)}{d}] + 1$$

where $[\alpha]$ denotes the integral part of α.